Input/Output Linearization Using Time Delay Control and Time Delay Observer

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In this paper, input/output linearization (IOL) method using time delay control (TDC) and time delay observer (TDO) is presented. This method enables the IOL method to be applied to plants even when all the states of plant are not measurable or the measured plant output is very noisy. The designed control system requires neither an accurate plant model nor the real time computation of plant nonlinearity. Consequently, the proposed control algorithm turned out to be computationally efficient and easy to design for nonlinear plants. In a simulation for a second order nonlinear plant, the output followed desired response well and the control performance appeared to be superior to IOL using TDC and numerical differentiation. Finally, in an experiment with a pneumatic servo system, we obtained results consistent with those from the simulation, and it was confirmed that the proposed control algorithm can be effectively used in a real closed-loop system.

Key Words: Nonlinear Observer, Time Delay Control, Differentiation, Diffeomorphism, Feedback Linearization, Decoupling, Transmission Zero

1. Introduction

Based on the concepts of differential geometry, the input-output linearization (IOL) method was developed as a base for nonlinear control system design (Isidori *et al.*, 1981; Isidori, 1985; Slotine and Li, 1991). This methods involve change of state variables into a normal form, disturbance decoupling, feedback linearization, and transmission zero. More specifically, state variable transformation is used in order to convert a nonlinear system to a linear one, and the problem is solved by feedback and local diffeomorphic state transformation. Besides it is noted that the problem of the IOL is closely related to the characteristics of transmission zero.

However, in order to apply the IOL methods to a real plant, the following three requirements must be considered: First, it is necessary to be able to measure all of the state variables and up to $(r-1)$-th time derivatives of the plant output, $y^{(r)}$, where r denotes the relative degree of the plant; second, an accurate plant model must be known; and third, real-time computation of the plant nonlinearities is required to obtain a linear input output behavior. Unfortunately, in engineering practice, there exist many cases that do not satisfy the measurability requirement. Furthermore, obtaining an accurate plant model is a time-consuming and complex procedure; and even if an accurate model is available, the burden for real-time computation of plant nonlinearities can be quite large depending on the complexity of plant nonlinearities. Hence, the three requirements present serious limitations on the implementations of IOL to real plants.

As an alternative approach to the conventional IOL, the IOL using time delay control (TDC) was proposed by Youcef-Toumi and Wu (1992). The simple algorithm neither uses state measurement, nor needs an accurate plant model, nor requires the real-time computation of plant non-

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linearities. Instead, it uses a time delay estimation of plant uncertainty: that is, using the plant input and up to r-th time derivatives of the plant output, the total plant uncertainties are estimated with very simple and efficient algorithm. Using this control method, a robust control performance against plant uncertainties can be obtained.

Nevertheless, although the IOL using TDC (IOLTDC) can alleviate the aforementioned requirements of state measurability, modeling, and real-time computation, one needs a way to estimate up to r-th derivatives of the plant output. In general, either additional sensors or numerical differentiators need to be used for the derivatives. However, the use of derivative sensors makes the overall system more complex and expensive, and the use of numerical differentiators makes the system more sensitive to measurement noise. Therefore, unless we find out an effective method to evaluate $y^{(r)}$, IOLTDC, a method addressing practical problems associated with IOL, can still remain impractical. Hence, an effective evaluation of $y^{(r)}$ is vital to real implementation.

In this paper, to overcome the problem of derivatives measurability in IOLTDC, we propose an approach to achieve IOL by using TDC and time delay observer (TDO). The TDO was introduced in Chang and Lee (1997) for nonlinear SISO plant in a phase variable form. Through this development, we want to make IOLTDC more applicable to real plants, and confirm that the original positive attributes of IOLTDC can still be preserved; such as simplicity, numerical efficiency, and robustness.

This paper is organized as follows. In the following section, the control problem is defined and the IOLTDC is reviewed. In section 3 the IOL method using TDC and TDO is proposed and its stability is discussed. In Sec. 4, a simulation study is undertaken to assure the validity of the proposed control algorithm. Sec. 5 presents the experimental results, followed by the conclusion in Sec. 6.

2. Input/Output Linearization Using Time Delay Control

To provide the basis for the development of this paper, the IOLTDC algorithm is briefly reviewed; more detailed exposition is found in Youcef-Toumi and Wu (1992). The nonlinear single-input, single-output (SISO) plant considered is described as follows:

$$\dot{\mathrm{x}}=\mathrm{f}(\mathrm{x})+\mathrm{g}(\mathrm{x})u,$$
$$y=h(\mathrm{x}), \tag{1}$$

where[1] $\mathrm{x}\in\Re^n$, $u\in\Re^1$, and $y\in\Re^1$ denote the state vector, the control input scalar, and the output scalar, respectively. The term $\mathrm{f}(\mathrm{x})$ represents the nonlinearities of the plant, $\mathrm{g}(\mathrm{x})$ the nonlinearities in the input, and $h(\mathrm{x})$ the nonlinear output distribution scalar. We assume that $\mathrm{f}(\mathrm{x})$, $\mathrm{g}(\mathrm{x})$, and $h(\mathrm{x})$ are sufficiently continuous functions of x.

In the input/output linearization procedure, the output is differentiated with respect to time several times until the control input u appears. Assume that r is the smallest integer such that the input appears in $y^{(r)}$, then

$$y^{(r)}=L_f{}^r(h(\mathrm{x}))+L_g(L_f{}^{r-1}(h(\mathrm{x})))u, \tag{2}$$

where $L_f(\cdot)$ and $L_g(\cdot)$ stand for the Lie derivative of $(\cdot)$ with respect to $\mathrm{f}(\mathrm{x})$ and $\mathrm{g}(\mathrm{x})$ respectively,

$$L_f{}^0(h(\mathrm{x}))=h(\mathrm{x}),$$
$$L_f{}^k(h(\mathrm{x}))=\left[\frac{\partial}{\partial\mathrm{x}}L_f{}^{k-1}(h(\mathrm{x}))\right]\mathrm{f}(\mathrm{x}), \tag{3}$$
$$L_g(L_f{}^k(h(\mathrm{x})))=\left[\frac{\partial}{\partial\mathrm{x}}L_f{}^k(h(\mathrm{x}))\right]\mathrm{g}(\mathrm{x}).$$

When the relative degree $r<n$, the nonlinear plant (1) can be transformed, using $\mathrm{z}=[y,\ \dot{y},\ \cdots,\ y^{(r-1)}]^T$ as a part of the new state components, into a normal form as,

$$\dot{\mathrm{z}}=\mathrm{E}_r\mathrm{z}+\mathrm{B}[a(\mathrm{x})+b(\mathrm{x})u], \tag{4}$$
$$y=\mathrm{C}\mathrm{z},$$
$$\dot{\eta}=\omega(\mathrm{x}) \tag{5}$$

with

[1] x, y, u, and all other variables derived from these are functions of time, t. For instance, $\mathrm{x}=\mathrm{x}(t)$.

$$\mathrm{z}=\begin{Bmatrix} y \\ \vdots \\ y^{(r-1)} \end{Bmatrix},\ \mathrm{E}_r=\begin{bmatrix} 0_{(r-1)\times 1} \vdots \mathrm{I}_{(r-1)} \\ 0_{1\times r} \end{bmatrix},$$

$$\mathrm{B}=\begin{bmatrix} 0_{(r-1)\times 1} \\ 1 \end{bmatrix},\ \mathrm{C}=[1 \vdots 0_{1\times(r-1)}], \tag{6}$$

$$a(\mathrm{x})=L_f{}^r(h(\mathrm{x})),\ b(\mathrm{x})=L_g(L_f{}^{r-1}(h(\mathrm{x}))),$$

where $\mathrm{z}\in\Re^r$ and $\eta\in\Re^{n-r}$ represent an external part and internal part of the plant dynamics (1), respectively. Note that the subsystem in a phase variable form (4) is simply another expression of (2), while the subsystem (5) does not contain the plant input

2.1 Control law

The first step in the IOLTDC design is to select a reference model such that the external part of plant (4), exhibits desirable linear behavior. In the context of model reference control, let the desired performance be specified by means of the response of a stable linear time-invariant reference model as

$$\dot{\mathrm{z}}_m=\mathrm{A}_m\mathrm{z}_m+\mathrm{B}_m r, \tag{7}$$

with

$$\mathrm{A}_m=\begin{bmatrix} 0_{(r-1)\times 1} \vdots \mathrm{I}_{r-1} \\ \mathrm{A}_{mr} \end{bmatrix},\ \mathrm{B}_m=\begin{bmatrix} 0_{(r-1)\times 1} \\ b_m \end{bmatrix}, \tag{8}$$

$$\mathrm{A}_{mr}=[a_{m1}, a_{m2}, \cdots, a_{mr}],$$

where $\mathrm{z}_m\in\Re^r$ denotes the state vector of the reference model, $r\in\Re^1$ the command scalar, A_m the system matrix, and B_m the command distribution vector.

Then the control objective is to find a control input u that makes the state of the plant asymptotically track the response of the reference model, (7). In other words, the tracking error $\varepsilon\equiv\mathrm{z}_m-\mathrm{z}$ is desired to satisfy the following error dynamics,

$$\dot{\varepsilon}=\mathrm{A}_m\varepsilon. \tag{9}$$

To find such a control law, subtract (4) from (7), then the control input results as follows:

$$u=b(\mathrm{x})^{-1}[-a(\mathrm{x})+\mathrm{A}_{mr}\mathrm{z}+b_m r], \tag{10}$$

which makes the tracking error satisfy (9), thereby achieving the control objective. If, in addition, the internal dynamics of (5) is exponentially stable and the reference input r is bounded, then the control input u is bounded.

The controller using (10) works only when $a(\mathrm{x})$ and $b(\mathrm{x})$ are known exactly. If there are uncertainties in the model, the system can no longer have linearized error dynamics as in (9). To overcome the problem, Youcef-Toumi and Wu (1992) proposed IOLTDC which achieves input/output linearization of plant without the exact knowledge of $a(\mathrm{x})$, given bound of $b(\mathrm{x})$ in (4) and relative degree indices r.

The IOLTDC adopts a particularly efficient estimation method based on the following idea (Youcef-Toumi and Wu, 1992; Youcef-Toumi and Ito, 1990; Hsia and Go, 1990): Firstly, since $a(\mathrm{x})$ is assumed as a continuous function, from which it follows that, for a sufficiently small time delay L,

$$a(\mathrm{x})\approx a(\mathrm{x}_{(t-L)}). \tag{11}$$

Secondly, use (4) and (11). Then, one can obtains the following estimation for $a(\mathrm{x})$.

$$\begin{aligned} a(\mathrm{x}) &= \dot{z}_r-b(\mathrm{x})u \\ &\approx \dot{z}_{r(t-L)}-b(\mathrm{x}_{(t-L)})u_{(t-L)}. \end{aligned} \tag{12}$$

Substituting this approximate estimation with $\hat{b}$ into (10) leads to the following control law:

$$u=\hat{b}^{-1}[-\dot{z}_{r(t-L)}+\hat{b}u_{(t-L)}+\mathrm{A}_{mr}\mathrm{z}+b_m r], \tag{13}$$

where $\hat{b}$ is a constant to be determined.

It is noteworthy that the control law, not requiring an accurate plant model nor the real-time computation of nonlinear dynamics of the plant, is quite simple and computationally efficient.

3. Input/Output Linearization using Time Delay Control and Time Delay Observer

As is clearly shown in (13), the IOLTDC requires the estimation of state, z and $\dot{z}_r$, which is the r-th derivative of the output, y. In practice, this requirement sets nontrivial limitations on the application of the controller to real plants. As a solution to this problem, use of the TDO, which is firstly introduced in Chang and Lee (1997) for nonlinear SISO plants in phase variable form, may be considered.

In this section, after designing the TDO for the external part of the plant, (4), we present the IOL method using TDC and TDO. In addition, the stability of resulting system consisting of the plant, controller and observer is discussed.

3.1 Observer design

Suppose that for the external part of the plant, (4), an observer of the following form is available:

$$\dot{\hat{z}} = E_r\hat{z} + B[a(x) + b(x)u] + K(C\hat{z} - y), \tag{14}$$

where $\hat{z} \in \Re^r$ denotes the reconstructed state vector, and $K \in \Re^{r\times 1}$ the observer gain matrix. Then the observation error $\hat{e} \equiv \hat{z} - z$ has an exponentially convergent dynamics as follows:

$$\dot{\hat{e}} = (A + KC)\hat{e}, \tag{15}$$

where an arbitrary convergence speed can be achieved by a suitable choice of $\mathbf{K}$.

In order to realize the observer, one must be able to estimate the uncertainty, $a(x)$ and $b(x)$. To this end, we have adopted from TDC (Youcef-Toumi and Ito, 1990; Hsia and Gao, 1009) the time delay estimation method in (12) using reconstructed state $\mathbf{z}$ and an approximate estimation with $\bar{b}$. Then, the resulting TDO law is as follows,

$$\dot{\hat{z}} = E_r\hat{z} + \alpha B[\dot{\hat{z}}_{r(t-L^*)} - \bar{b}u_{(t-L^*)}] + K(C\hat{z} - y), \tag{16}$$

where L^* is the time delay for TDO, and α is a constant to be determined.

Note that the design of TDO in (16) does *not* require an accurate plant model nor the computation of nonlinearities $a(x)$ and $b(x)$. Therefore, the algorithm of the TDO is very simple and computationally efficient. When the Euler integration method is used for digital implementation, the evaluation of (16) requires $(3r+2)$ additions and $(2r+3)$ multiplications in each sampling interval; the computational efficiency of the TDO is better than that of many other nonlinear observers (Misawa and Hedrick, 1989).

The idea behind introducing the constant α is based on the attempt by Youcef-Toumi and Wu (1992). That is, introducing α has an effect of using a low pass filter. Increasing α close to 1 improves the performance robustness to plant uncertainty, but makes the observer more sensitive to measurement noise; while decreasing α reverses the balance. Therefore, a careful tuning of α is required for a good compromise between robustness to uncertainty and sensitivity to measurement noise. This point was explained in more detail in Chang and Lee (1997).

When the TDO (together with TDC) is connected to the plant, the control input is to be obtained by using the reconstructed state $\hat{z}$ instead of the state z. Thus, the control input u is determined as

$$u = \bar{b}^{-1}[-\dot{\hat{z}}_{r(t-L)} + \bar{b}u_{(t-L)} + A_{mr}\hat{z} + b_m r] \tag{17}$$

3.2 Stability of overall system

In this section, the stability of resulting system consisting of the plant (4), the TDC (17), and the TDO (16) is analyzed. This analysis provides a sufficient condition for the stability of the overall system. Thus if the TDC and TDO are designed so that the observer parameters K, α, and L^*, and controller parameters, A_m, B_m, L, and $\bar{b}$ meet this condition, then the resulting system is made stable.

To the external part of plant dynamics (4), the following model properties are inherent and are useful in the subsequent section for the stability analysis.

Lemma 1 : If the plant under consideration and its controller and observer are well defined over the interval $0 \le t \le T$, then $a(x)$ and $b(x)$ in (4) are uniformly continuous functions of time for $0 \le t \le T$. Therefore, as time delay L is sufficiently small, the values of $a(x_{(t-L)})$, $b(x_{(t-L)})$ and $u_{(t-L)}$ will converge to $a(x)$, $b(x)$ and u. That is $a(x_{(t-L)}) = a(x) + O_a(L)$, $b(x_{(t-L)}) = b(x) + O_b(L)$ and $u_{(t-L)} = u + O_u(L)$.

Proof : This lemma is identical to the Lemma 1 of Youcef-Toumi and Wu (1992), and the proof is clearly shown in Youcef-Toumi and Wu (1992).

In the following two lemmas, the observation error equation and model following error equation are derived as the functions of new observation error vector $\tilde{e}$ and new model following error vector $\tilde{\varepsilon}$: One can find similar procedures for derivation in Chang and Lee (1997).

$$\tilde{\mathrm{e}}=\begin{bmatrix}\hat{\mathrm{e}}\\ \cdots \\ \tilde{e}_{r+1}\end{bmatrix} \text{ with } \tilde{e}_{r+1}=\dot{e}_r,\ \tilde{\varepsilon}=\begin{bmatrix}\varepsilon\\ \cdots \\ \tilde{\varepsilon}_{r+1}\end{bmatrix} \text{with } \tilde{\varepsilon}_{r+1}=\dot{\varepsilon}_r. \tag{18}$$

Lemma 2 : If the observer time delay L^* is sufficiently small, then the resulting observation error equation can be expressed as

$$\dot{\tilde{\mathrm{e}}}=\mathrm{A}_1\tilde{\mathrm{e}}+\mathrm{A}_2\tilde{\varepsilon}+\mathrm{f}(\dot{z}_m),+\mathrm{O}_{\tilde{e}}(L) \tag{19}$$

with

$$\mathrm{A}_1=\begin{bmatrix} -K_1 & 1 & 0 & \cdots & 0\\ -K_2 & 0 & 1 & \cdots & 0\\ \vdots & \vdots & & & \vdots\\ 0 & 0 & 0 & \cdots & 1\\ -\frac{K_r}{\alpha L} & 0 & \cdots & 0 & -\frac{(1-\alpha)}{\alpha L}\end{bmatrix},$$

$$\mathrm{A}_2=\begin{bmatrix}0 & \cdots & 0\\ \vdots & & \vdots\\ 0 & \cdots & \frac{(1-\alpha)}{\alpha L}\end{bmatrix},\ \mathrm{f}(\dot{z}_m)=\begin{bmatrix}0_{r\times 1}\\ \cdots\\ -\frac{(1-\alpha)}{\alpha L}\dot{z}_{mr}\end{bmatrix},$$

$$\mathrm{O}_{\tilde{e}}(L)=\begin{bmatrix}0_{r\times 1}\\ \cdots\\ \frac{1}{L}[O_a(L)+O_b(L)(O_u(L)+u_{(t-L)})]-\\ O_{\tilde{e}}^2(L)-\frac{\hat{b}}{\alpha L}O_u(L)\end{bmatrix}$$

with $O_{\tilde{e}}^2(L)=\frac{\dot{e}_r-\dot{e}_{r(t-L)}}{L}-\ddot{e}_r$.

Lemma 3 : TDC in (17), if used for the plant (4) with sufficiently small time delay L, has the following model following error equation:

$$\dot{\tilde{\varepsilon}}=\mathrm{A}_3\tilde{\mathrm{e}}+\mathrm{A}_4\tilde{\varepsilon}+\mathrm{O}_{\tilde{\varepsilon}}(L) \tag{20}$$

with

$$\mathrm{A}_3=\begin{bmatrix}0_{r\times(r+1)}\\ \cdots\\ \Sigma\end{bmatrix},\ \mathrm{A}_4=\begin{bmatrix}0_{r\times 1} : \mathrm{I}_r\\ \cdots\\ \Sigma\end{bmatrix},$$

$$\mathrm{O}_{\tilde{\varepsilon}}(L^*)=\begin{bmatrix}0_{r\times 1}\\ \cdots\\ \frac{1}{L^*(1-b(\mathrm{x})\hat{b}^{-1})}[O_b(L^*)u_{(t-L^*)}+O_a(L^*)\\ -b(\mathrm{x})\hat{b}^{-1}O_{\tilde{e}}(L^*)]-O_\varepsilon^2(L^*)\end{bmatrix}$$

with,

$$\Sigma=[Ga_{m1}, Ga_{m2}, \cdots, Ga_{mr}, -G],$$
$$G=\frac{b(\mathrm{x})\hat{b}^{-1}}{L^*(1-b(\mathrm{x})\hat{b}^{-1})},$$
$$O_{\tilde{e}}(L^*)=\dot{e}_{r(t-L^*)}-\dot{e}_r,$$
$$O_\varepsilon^2(L^*)=\frac{\dot{\varepsilon}_r-\dot{\varepsilon}_{r(t-L^*)}}{L^*}-\ddot{\varepsilon}_r.$$

From Lemma 1 and Lemma 2, system error Eqs. (19) and (20) yield

$$\begin{Bmatrix}\dot{\tilde{\mathrm{e}}}\\ \dot{\tilde{\varepsilon}}\end{Bmatrix}=\begin{bmatrix}\mathrm{A}_1 & \mathrm{A}_2\\ \mathrm{A}_3 & \mathrm{A}_4\end{bmatrix}\begin{Bmatrix}\tilde{\mathrm{e}}\\ \tilde{\varepsilon}\end{Bmatrix}+\begin{Bmatrix}\mathrm{f}(\dot{z}_m)\\ 0\end{Bmatrix}+\begin{Bmatrix}\mathrm{O}_{\tilde{e}}(L)\\ \mathrm{O}_{\tilde{\varepsilon}}(L^*)\end{Bmatrix}, \tag{21}$$

which can be rewritten as

$$\dot{\mathrm{e}}_{ol}=\mathrm{A}_{ol}\mathrm{e}_{ol}+\mathrm{f}_{ol}(\dot{z}_m)+\mathrm{O}_{ol}(L, L^*), \tag{22}$$

where $\dot{\mathrm{e}}_{ol}=[\tilde{\mathrm{e}}^T, \tilde{\varepsilon}^T]^T$. The stability of a system of the form given in (22) is summarized in Theorem 1.

Theorem 1 : Consider the system in (22) under the following assumptions:

A1) All the eigenvalues of A_{ol} are different from zero and have a negative real part:

A2) $\|\mathrm{f}_{ol}(\dot{z}_m)\|<a_1e^{-Kt}$,

A3) $\|\mathrm{O}_{ol}(L, L^*)\|<\mu$ with zero steady state value.

Then $\|\mathrm{e}_{ol}\|$ is exponentially convergent to the open domain

$$\Omega(\delta)=\{\mathrm{e}_{ol} : 0\le\|\mathrm{e}_{ol}\|<\delta\}, \tag{23}$$

with $\delta=2p_0(a_1e^{-Kt}+\mu)$, where p_0 is an upper bound on the norm of the symmetric and positive definite matrix $\mathbf{P}$ obtained from the solutions of $\mathrm{A}_{ol}^T\mathrm{P}+\mathrm{P}\mathrm{A}_{ol}=-\mathrm{I}$, where $\|\mathrm{P}\|\le p_0$. Furthermore, as the time goes to infinity $\|\mathrm{e}_{ol}\|$ goes to zero.

Proof : Define a Lyapunov function as $V=\mathrm{e}_{ol}^T\mathrm{P}\mathrm{e}_{ol}$ with $\mathrm{P}=\mathrm{P}^T$ and $\|\mathrm{P}\|\le p_0$. Using the assumptions A1) and A2), $\dot{V}$ can be obtained as follows:

$$\begin{aligned}\dot{V}&=\mathrm{e}_{ol}^T[\mathrm{A}_{ol}^T\mathrm{P}+\mathrm{P}\mathrm{A}_{ol}]\mathrm{e}_{ol}+2\mathrm{e}_{ol}^T\mathrm{P}\mathrm{f}_{ol}(\dot{z}_m)\\ &\quad+2\mathrm{e}_{ol}^T\mathrm{P}\mathrm{O}_{ol}(L, L^*)\\ &\le-\|\mathrm{e}_{ol}\|^2+2p_0(a_1e^{-Kt}+\mu)\|\mathrm{e}_{ol}\|.\end{aligned} \tag{24}$$

Hence, for any $\|\mathrm{e}_{ol}\|$ that satisfies $\|\mathrm{e}_{ol}\|\ge 2p_0(a_1e^{-Kt}+\mu)=\delta$, $\dot{V}$ remains negative, causing $\|\mathrm{e}_{ol}\|$ to converge to $\Omega(\delta)$.

In addition, by the assumptions A2) and A3), δ decreases to zero in steady state. Therefore, as the time goes to infinity $\|\mathrm{e}_{ol}\|$ goes to zero. (Q.E.

D.)

Remarks :

1) This theorem provides a sufficient condition for the stability of overall system. Thus if the proposed control system is designed so that the TDO gains K and α, TDC gains A_m, B_m and $\hat{b}$, and time delay L and L^* meet the condition of A1) and A2), then the resulting system is made stable.

2) For a stable system matrix A_m, the reference input r that satisfies the condition of A2) includes impulse inputs, step inputs, and all of bounded continuous inputs with constant steady state value. An example of the reference input that does not satisfy the condition of A2) is sinusoidal inputs.

3) For sufficiently small time delays L and L^*, the magnitude of $O_{ol}(L, L^*)$ is very close to zero and has zero steady state value.

4) The size of δ can be reduced by a suitable choice of the TDO gains, the TDC gains, and time delays L and L^*.

4. Simulation

In order to demonstrate the effectiveness of the proposed control system, a third order system with stable zero dynamics is simulated. The system is described in Slotine and Lee (1991) and rewritten below,

$$\dot{\mathrm{x}}=\begin{bmatrix} x_3-x_2^3 \\ -x_2 \\ x_1^2-x_3 \end{bmatrix}+\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}u,\ y=x_1. \tag{25}$$

For this system, by defining new variable z_1,z_2, and η as

$$\begin{aligned} z_1&=x_1=y, \\ z_2&=x_3-x_2^3, \\ \eta&=x_2+x_3, \end{aligned} \tag{26}$$

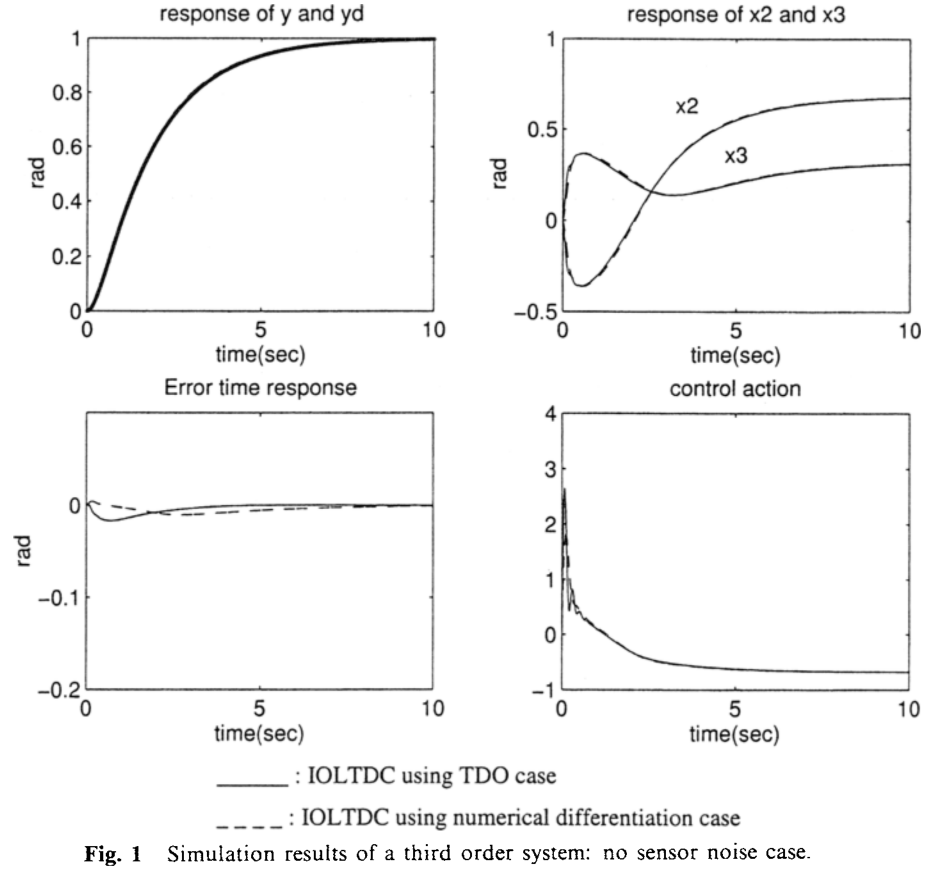


Fig. 1 Simulation results of a third order system: no sensor noise case.

we obtain equations of the form of (4) and (5) as follows:

$$\begin{aligned} \dot{z}_1 &= z_2, \\ \dot{z}_2 &= a(\mathrm{x}) + b(\mathrm{x})\, u, \\ \dot{\eta} &= z_1^2 - z_3, \end{aligned} \tag{27}$$

where

$$\begin{aligned} a(\mathrm{x}) &= L_f^2 h(\mathrm{x}) = x_1^2 + 3x_2^3 - x_3, \\ b(\mathrm{x}) &= L_g L_f h(\mathrm{x}) = 1 + 3x_2^3. \end{aligned} \tag{28}$$

Note that the zero dynamics, from (27), is exponentially stable. Applying (16) and (17), the following TDO and TDC are obtained:

$$\begin{aligned} \dot{\hat{z}}_1 &= \hat{z}_2 + K_1(\hat{z}_1 - y), \\ \dot{\hat{z}}_2 &= \alpha[\dot{\hat{z}}_{2(t-L^*)} - \hat{b}u_{(t-L^*)} + \hat{b}u] \\ &\quad + K_2(\hat{z}_1 - y), \end{aligned} \tag{29}$$

$$\begin{aligned} u = \hat{b}^{-1}[&-\dot{\hat{z}}_{2(t-L)} + \hat{b}u_{(t-L)} + a_{m1}\hat{z}_1 \\ &+ a_{m2}\hat{z}_2 + b_m r]. \end{aligned} \tag{30}$$

In the simulations, the parameter of the TDO and TDC are determined as $K_1 = -100$, $K_2 = -1000$, $\alpha = 0.8$, $a_{m1} = -4$, $a_{m2} = -2$, $\hat{b} = 4$, and the time delays, L, and L^* are set at $0.01s$ and $0.001s$, respectively. The selected parameters satisfy the stability condition of Theorem 1.

In the simulations, we will show that the proposed IOLTDC using TDO can indeed achieve satisfactory control performance compared with the control system using numerical differentiation with low-pass filter. In numerical differentiation case, the cut-off frequency of the low-pass filter is set at 100 *rad/s*.

Figures 1 and 2 show the responses of two control systems in two different cases: without measurement noise and with measurement noise (random noise with magnitude of 0.1 is added to the measurement of y). As shown in Fig. 1, when the measurement noise does not exist, the two control systems follow the desired response y_d well and other two states x_2 and x_3 are bounded. However, the measurement noise in the plant

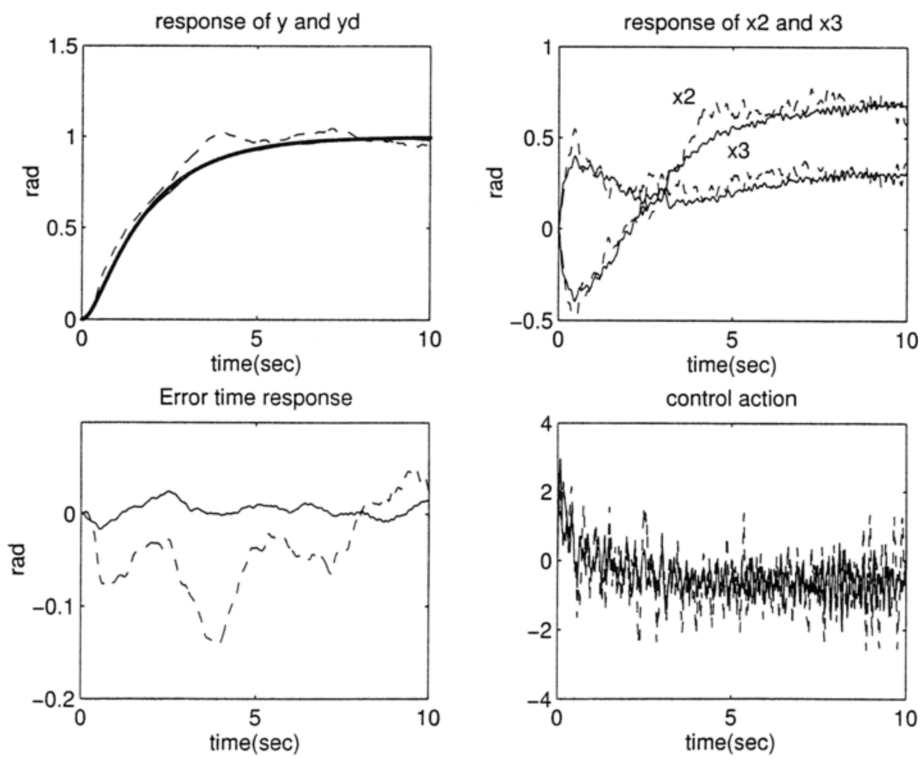


Fig. 2 Simulation results of a third order system: with sensor noise case ($\sigma = 0.001$).

output results in sluggish responses with the control system using numerical differentiation (Fig. 2). By comparison, the proposed control system performs well and the measurement noise does not seriously degrade the control performance. So it is confirmed that the proposed control system is less sensitive to measurement noise than the control system using numerical differentiation with low-pass filter.

5. Experiments

In order to test performances of the proposed control system in practical circumstances, the proposed algorithm is applied to the position control of a pneumatic cylinder system. The schematic diagram of pneumatic system is shown in Fig. 3, and the model is shown in Liu and Bobrow (1988).

In this experiment, since the relative degree of the plant is 3, the IOLTDC requires velocity, acceleration, and jerk to be reconstructed from the position measurement. The purpose of this experiment is threefold: to show that the proposed control system can be readily applied to the practical areas where plant dynamics is highly nonlinear; to investigate on how it compares with another method that does not require a model, numerical differentiation with low-pass filter, which is also frequently used in practice; and to compare the control performance of the proposed control system with a well tuned PID control, the control performance of which is widely recognized.

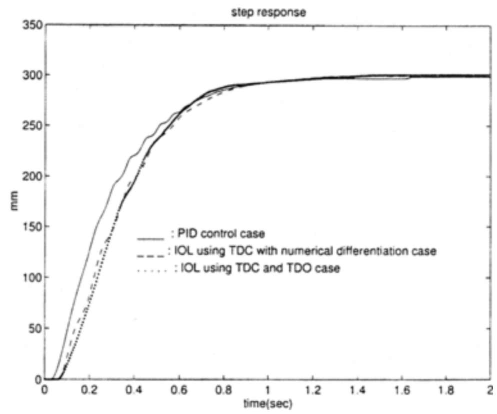


Fig. 4 Experimental results of three control systems when step input is applied (with a piston mass of 500 g).

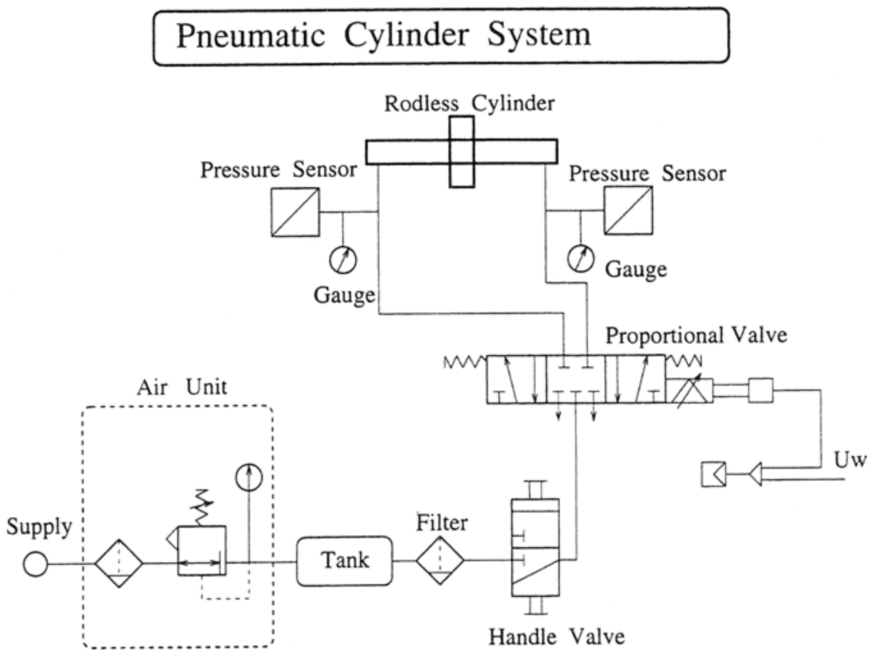


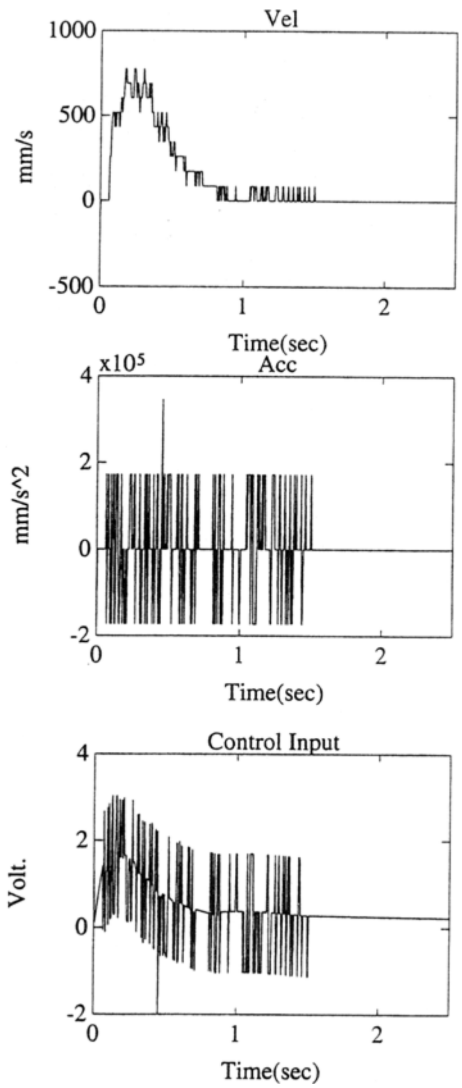
Fig. 3 Schematic diagram of pneumatic cylinder system.

The system consists of the following components: a rodless cylinder having a stroke of 600 *mm*, a piston mass of 500 *g*, a diameter of 40 *mm*, and an operating pressure of 5 *bar* (gauge); a flow control valve with a nominal flow rate of 700 *l/min*; a rotary encoder with a timing belt that can measure linear displacement with a resolution of 0.04313 *mm/pulse*; and a DSP board where the controller and observer are implemented.

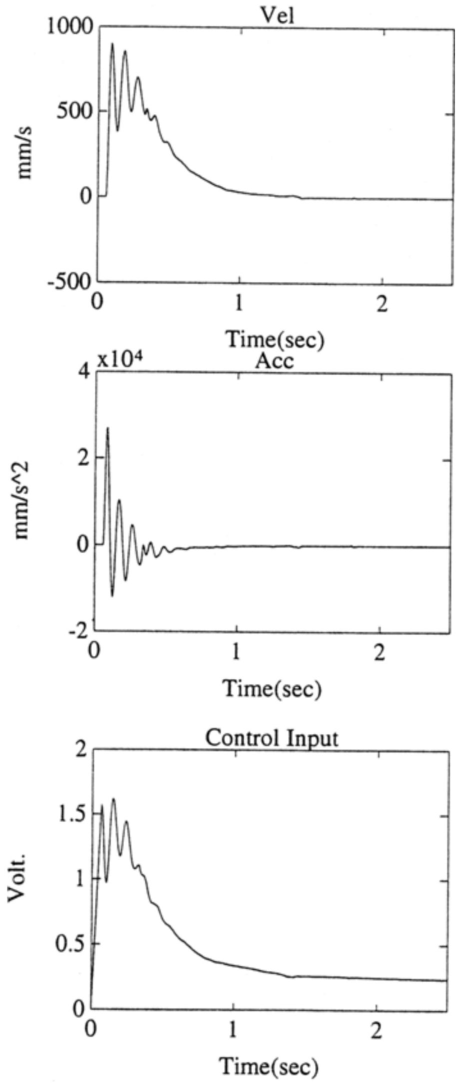
TDO was implemented with a sampling frequency of 2000 *Hz*. For numerical differentiation used with IOLTDC, the backward difference method was incorporated together with a low-pass filter, the cut-off frequency of which was carefully tuned — otherwise, the system became unstable — to be about 80 *rad/s*. The reference model is given as

$$\dot{\mathrm{x}}_m = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -10000 & -3500 & -120 \end{bmatrix} \mathrm{x}_m + \begin{bmatrix} 0 \\ 0 \\ 10000 \end{bmatrix} r.$$

Figure 4 shows the responses to the step input of $r=300$ *mm* when the mass of piston is 500 *g*



(a) Using numerical differentiator



(b) Using TDO

Fig. 5 Experimental results of three control systems when step input is applied (with a piston mass of 500 *g*).

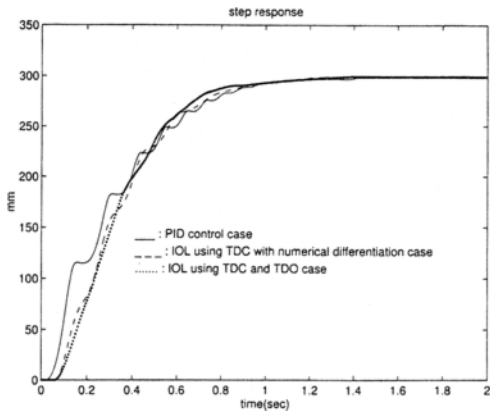


Fig. 6 Experimental results of three control systems when step input is applied (with a piston mass of 1000 g).

in three different cases: with IOLTDC using TDO, with IOLTDC using numerical differentiation, and with well tuned PID controller. In addition, Fig. 5 shows velocity, acceleration, and jerk, reconstructed with numerical differentiation and TDO, respectively. As shown in Figs. 4 and 5, under a nominal mass of 500 g, one has similar performances with the three control schemes, yet slightly better results with PID than others in terms of rising time. The comparison of the reconstructed signals confirms that the TDO reconstructs the states smoothly and stably. By comparison, the numerical differentiation is observed to amplify the sensor noise to a substantial degree.

Figure 6 shows responses of three control systems when the mass of piston is increased to 1000 g. As the piston mass increases, the performance with PID control system becomes worse — revealing more oscillatory response. In contrast, the other schemes using TDC show little oscillation, thereby demonstrating the robustness of IOLTDC to parameter variations.

6. Conclusion

In this paper, we have developed IOLTDC using TDO. As to the resulting system consisting of TDC and TDO, the stability of the overall system is analyzed and it is discussed how to determine the parameters of the resulting control system. It is observed that the proposed control algorithm, not requiring the plant model and real-time computation, has very simple structure and is numerically efficient.

The effectiveness of the proposed control system has been evaluated through a simulation example on a second order nonlinear plant. Simulation results show that with the IOLTDC using TDO, the plant output is made to track desired response y_d quite well, and the proposed control system is less sensitive to sensor noise than the IOLTDC with numerical differentiation. An experiment on a pneumatic servo system shows results consistent with those obtained from the simulation. In addition, it turns out that the proposed control system was shown to be quite robust to piston mass variation, implying that the robustness of TDO is preserved. With the proposed method, therefore, IOLTDC becomes more applicable to real plants when the measured output is very noisy.

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